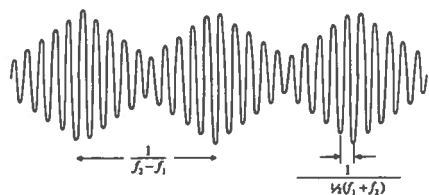


### Demonstration 32. Primary and Secondary Beats. (1:32)

If two pure tones have slightly different frequencies  $f_1$  and  $f_2 = f_1 + \Delta f$ , the phase difference  $\phi_2 - \phi_1$  changes continuously with time. The amplitude of the resultant tone varies between  $A_1 + A_2$  and  $A_1 - A_2$ , where  $A_1$  and  $A_2$  are the individual amplitudes. These slow periodic variations in amplitude at frequency  $\Delta f$  are called *beats*, or perhaps we should say *primary beats*, to distinguish them from second-order beats, that will be described in the next paragraph. Beats are easily heard when  $\Delta f$  is less than 10 Hz, and may be perceived up to about 15 Hz.

A sensation of beats also occurs when the frequencies of two tones  $f_1$  and  $f_2$  are nearly, but not quite, in a simple ratio. If  $f_2 = 2f_1 + \delta$  (mistuned octave), beats are heard at a frequency  $\delta$ . In general, when  $f_2 = (n/m)f_1 + \delta$ ,  $m\delta$  beats occur each second. These are called *second-order beats* or *beats of mistuned consonances*, because the relationship  $f_2 = (n/m)f_1$ , where  $n$  and  $m$  are integers, defines consonant musical intervals, such as a perfect fifth ( $3/2$ ), a perfect fourth ( $4/3$ ), a major third ( $5/4$ ), etc.



Waveform with beats due to pure tones with frequencies  $f_1$  and  $f_2 = f_1 + \Delta f$ .

(from Rossing, 1982)

Primary beats can be easily understood as an example of linear superposition in the ear. Second-order beats between pure tones are not quite so easy to explain, however. Helmholtz (1877) adopted an explanation based on combination tones (Demonstration 34), but an explanation by means of aural harmonics was favored by others, including Wegel and Lane (1924). This theory, which explains second-order beats as resulting from primary beats between aural harmonics of  $f_1$  and  $f_2$ , predicts the correct frequency  $mf_2 - nf_1$ , but cannot explain why the aural harmonics themselves are not heard (Lawrence and Yantis, 1957).

An explanation which does not require nonlinear distortion in the ear is favored by Plomp (1966) and others. According to this theory, the ear recognizes periodic variations in waveform, probably as a periodicity in nerve impulses evoked when the displacement of the basilar membrane exceeds a critical value. This implies that simple tones can interfere over much larger frequency differences than the critical bandwidth, and also that the ear can detect changing phase (even though it is a poor detector of phase in the steady state).

Beats of mistuned consonances have long been used by piano tuners, for example, to tune fifths, fourths, and even octaves on the piano. Violinists also make use of them in tuning their instruments. In the case of musical tones, however, primary beats between harmonics occur at the same rate as second-order beats, and the two types of beats cannot be distinguished.

In the first example, pure tones having frequencies of 1000 and 1004 Hz are presented together, giving rise to primary beats at a 4-Hz rate.

In the next example, tones with frequencies of 2004 Hz, 1502 Hz, and 1334.67 Hz are combined with a 1000-Hz tone to give secondary beats at a 4-Hz rate ( $n/m = 2/1$ ,  $3/2$ , and  $4/3$ , respectively).

It is instructive to compare the apparent strengths of the beats in each case.

### Commentary

"Two tones having frequencies of 1000 and 1004 Hz are presented separately and then together. The sequence is presented twice."